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LONGITUDINAL SPACE CHARGE FORCES WITHIN A BUNCHED BEAM IN THE PRESENCE OF MAGNETIC LAMINATION

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1. INTRODUCTION

This paper deals with the computation of the longitudinal electric field within a bunched beam in the special case that the bending magnet gaps are used as vacuum tanks and the closest items to the beam are the pole surfaces.[1]

The magnet lamination is taken into account by assimilating each lateral crack to a radial transmission line.

Numerical results, applied to the case of the NAL Booster, show that the usual bunch wake field with uniform resistive wall are considerably smaller than the field in the presence of the magnet lamination, so that it can be neglected at any energy and frequency of interest. Comparison of the lamination wake field to the "reactive" beam-wall field shows that the former is negligible at injection, gives substantial contribution at transition and is predominant at ejection.

2. THE METHOD

The Fourier transform of the longitudinal electric field within a particle bunched beam takes the following form,[2] at the angular frequency ω equal to $\underline{n}$ times the bunch angular velocity Ω_0,

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$$E_n = -C_{bw} \frac{\partial \lambda_n}{\partial z} - Z_n' \ I_n \qquad (1)$$

where λ_n and I_n are the Fourier transforms of the beam charge per unit length and the beam current

$$\lambda_n = Ne \ f_n \ e^{i(kz-\omega t)}, \ k = n/R$$

$$I_n = \beta c \ \lambda_n$$

N, number of particles within the bunch

e, particle charge

c, light velocity

$v = \beta c$, bunch velocity

R, bunch orbit radius

f_n, Fourier transform of the longitudinal particle distribution function, normalized to unity

z, t, longitudinal coordinate and time.

C_{bw} is the remaining capacitance per unit length between beam and surrounding wall after the magnetic cancellation. For circular geometry it is

$$C_{bw} = (1-\beta^2) \ (1+2 \ \ln \frac{b}{a})$$

with

a, beam radius

b, radius of the surrounding wall.

Z_n' is the impedance per unit length characteristic of the media surrounding the beam at the boundary.

We need computation of the equivalent Z_n' for the Booster magnet lamination.

We shall consider circular geometry as sketched in Figure 1, where the relative dimensions have been exaggerated with respect to the practice case.

The symbols b_1, b_2, D, d and Δ are self-explanatory by inspecting the Figure 1.

We assume that vacuum is in the cavity at the center as well as in the lateral cracks.

The wall is of conductive material characterized by high conductivity σ and magnetic permeability μ, and by standard dielectric constant ε. The material holds the following characteristic surface impedance

$$\zeta = (1-i)\,\mathcal{R}$$

with $\mathcal{R}$ the material resistivity

$$\mathcal{R} = \sqrt{\frac{\omega\mu}{8\pi\sigma}}$$

valid for $\omega \ll 4\pi\sigma/\varepsilon$.

If the skin depth δ characteristic of the wall material at the angular frequency ω

$$\delta = c/(2\pi\mu\omega\sigma)^{1/2}$$

is enough smaller (but not necessarily much smaller) than the lamination sheet length D and the material thickness Δ around the cylindric structure, the equivalent impedance per unit length Z_n' is defined by inspecting the Figure 2.

The current I flows at the surface of the metallic boundary through a depth of length δ. In this situation the electric and magnetic fields do not propagate to ground, which, by definition, is the outer conductive surface of the cylinder. Thus the lamination structure can be outlined by an equivalent electric series circuit of impedances Z_W and Z_c alternately and uniformly distributed along the longitudinal direction.

Z_w is the equivalent sheet impedance, i.e. between the points A, B or C, D as shown in Figure 2. It depends only on the material resistivity

$$Z_w = \frac{2\zeta}{cb_1} D. \tag{2}$$

Z_c is the equivalent crack impedance, i.e. between the points B, C as shown in Figure 2. Thus the equivalent lamination impedance per unit length Z_n' can be

$$Z_n' = \frac{Z_w + Z_c}{D+d} . \tag{3}$$

To work out Z_c we assimilate the crack cavity to a radial transmission line.

3. THE CRACK TRANSMISSION LINE

Let us make reference to the Figure 3. The current I is passing the crack cavity only in the direction from A to B.

The element of cavity extending from r to r+dr can be replaced by the lumped circuit shown in the same Figure.

Introducing the crack capacitance and inductance per unit length, respectively,

$$C_u = \frac{r}{2d}$$

$$L_u = \frac{2d}{rc^2} \quad ,$$

we have

$$dY = -i \ \omega \ C_u \ dr$$

$$dZ = \left(\frac{2\zeta}{cr} - i \ \omega \ L_u\right) \ dr$$

For frequencies satisfying

$$\omega << \frac{1}{8\pi} \frac{\mu \ c^2}{\sigma \ d^2}$$

the crack inductive impedance $-i \ \omega \ L_u$ can be neglected with respect to the complex resistivity of the wall material.

In the following we shall denote the impedance and the admittance per unit length with

$$Z_u = (1-i) \ \frac{2\ell}{cr} \tag{4}$$

and

$$Y_u = -i \ \omega \ \frac{r}{2d} \tag{5}$$

respectively.

The crack cavity can be, then, replaced by the transmission line shown below in the Figure 3. This transmission line can be treated in terms of the following quantities:

V_{lo}, potential function at the lower crack surface

V_{up}, potential function at the upper crack surface

I_{lo}, current flowing on the lower crack surface

I_{up}, current flowing on the upper crack surface

V_c, voltage drop across the crack

j, current per unit length crossing the crack.

The following are the relationships binding them

$$V_c = V_{lo} - V_{up} \tag{6}$$

$$j = Y_u V_c \tag{7}$$

$$\frac{dI_{lo}}{dr} = -j \tag{8}$$

$$\frac{dI_{up}}{dr} = j \tag{9}$$

$$\frac{dV_{lo}}{dr} = -Z_u I_{lo} \tag{10}$$

$$\frac{dV_{up}}{dr} = -Z_u I_{up} \tag{11}$$

From these we derive the following second order differential equation in V_c

$$\frac{d^2V_c}{dr^2} - \frac{1}{Z_u}\frac{dZ_u}{dr}\frac{dV_c}{dr} - 2\ Y_u\ Z_u\ V_c = 0$$

which from (4) and (5) can be written also

$$\frac{d^2V_c}{dr^2} + \frac{1}{r}\frac{dV_c}{dr} + g^2\ V_c = 0 \tag{12}$$

with

$$g^2 = 2(1+i)\ \frac{\omega \mathcal{R}}{cd} = -2\ Y_u\ Z_u \tag{13}$$

The general solution of Eq. (12) is

$$V_c(r) = c_1\ J_0(gr) + c_2\ N_0(gr) \tag{14}$$

where J_m and N_m are the ordinary Bessel functions. c_1 and c_2 are two constants. These can be determined with the boundary conditions at $r = b_1$ and $r = b_2$.

4. THE BOUNDARY CONDITIONS

The crack transmission line is terminated at $r = b_2$ by the impedance Z_T which is simply given by the resistivity of the bottom material

$$Z_T = \frac{2\zeta}{cb_2} d.$$

This impedance is passed by the current

$$I_T = I_{lo}(r=b_2) = -I_{up}(r=b_2)$$

or, from (10) and (11)

$$I_T = -\frac{1}{2} \frac{dV_c}{dr} \Bigg|_{r=b_2} \Bigg/ Z_u(r=b_2)$$

with the voltage drop

$$V_c(r=b_2) = Z_T \, I_T.$$

Thus the boundary condition at $r=b_2$ is

$$V_c = -\frac{d}{2} \frac{dV_c}{dr} \quad , \; (r=b_2)$$

i.e. from Eq. (14),

$$c_1 \left[J_0(gb_2) - \frac{dg}{2} J_1(gb_2)\right] + c_2 \left[N_0(gb_2) - \frac{dg}{2} N_1(gb_2)\right] = 0 \tag{15}$$

The boundary condition at $r=b_1$ is

$$I_{lo} = I, \; I_{up} = -I, \quad (r=b_1)$$

from which and (6), (10) and (11) we derive

$$\frac{dV_c}{dr} = -2 \, Z_u I \, , \qquad (r=b_1)$$

i.e., from Eqs. (13) and (14),

$$c_1 J_1(gb_1) + c_2 N_1(gb_1) = 2\frac{Z_u(b_1)}{g} I \tag{16}$$

We solve the system of equations (15) and (16) for c_1 and c_2

$$c_1 = -2\frac{Z_u(b_1)}{g} \; \frac{N_0(gb_2) - \frac{dg}{2} N_1(gb_2)}{S} I$$

$$c_2 = 2\frac{Z_u(b_1)}{g} \; \frac{J_0(gb_2) - \frac{dg}{2} J_1(gb_2)}{S} I$$

with S the determinant of the system

$$S = J_0(gb_2)N_1(gb_1) - J_1(gb_1)N_0(gb_2) + \\ - \frac{dg}{2}\left\{ J_1(gb_2)N_1(gb_1) - J_1(gb_1)\, N_1(gb_2)\right\} .$$

The crack impedance takes the form

$$Z_c = \frac{c_1 J_0(gb_1) + c_2 N_0(gb_1)}{I} \tag{17}$$

5. APPLICATION TO THE NAL BOOSTER

In the Booster, the magnet lamination covers about α = 60% of the total ring circumference. We take

$D = 0.06$ cm

$d = 0.001$ cm

$b_1 = 3.$ cm

$b_2 = 15.$ cm

$\sigma = 5. \times 10^{16}$ sec^{-1}

$\mu = 100$

$a = 0.5$ cm

The cracks are covered by an insulator material of a dielectric constant $\varepsilon_c \sim 6$. This can be taken into account in our computation by simply multiplying the capacitance per unit length C_u per ε_c. The resistivity of the insulator is neglected.

The angular revolution frequency is

$$\Omega_0 = 4\ \beta\ \text{MHz}$$

and we limit our computation to the following range

$$0 \leqslant n \leqslant 1000$$

In this range the lateral wall impedance Z_w, Eq. (2), results be always very much smaller than the crack impedance Z_c, Eq. (17). Indeed, we obtain

$$Z_w = (1-i)\ 0.4 \times 10^{-15}, \text{ at } \omega = 10^9 \text{ sec}^{-1}$$

Since $d \ll D$, we replace Eq. (3) by the approximated one

$$Z_n' = \alpha \frac{Z_c}{D} \quad .$$

The real and imaginary parts of Z_n' have been plotted in Fig. 4 against ω. They correspond to the continuous curves.

In order to compare the first term to the second one at the r.h. side of Eq. (1), we rewrite

$$E_n = -(Z_{bw}' + Z_n')\ I_n$$

with

$$Z_{bw}' = i\ \frac{nC_{bw}}{\beta c R}$$

Z_{bw}' has been plotted in Fig. 4 at three instants of the acceleration cycle.

The total longitudinal electric field within the bunched beam is obtained by anti-transforming (1), i.e.

$$E = \sum_{n=-\infty}^{+\infty} E_n$$

For numerical computation purposes we approximate the above summation by the following

$$E = \sum_{n=-1000}^{+1000} E_n$$

and we take a parabolic particle distribution. The results are shown in Fig. 5. Bunch internal coordinate, x, is in the abscissa and longitudinal electric field per unit charge in the ordinate. The continuous straight lines show the reactive beam-wall field, corresponding to the first term at the r.h. side of Eq. (1), and the dashed lines the wake field. The bunch half-length L/2 and the γ used are beside each picture.

REFERENCES

[1]This work was done independently and contemporaneously to the work done by S. C. Snowdon on the same topic. Although the methods of approach to the problem were different, they gave the same result when applied to the NAL Booster.
S. C. Snowdon, "Wave Propagation Between Booster Laminations Induced by Longitudinal Motion of Beam," TM-277 (0300), (November, 1970).

[2]A. G. Ruggiero, "Longitudinal Space Charge Forces within Bunched Beams," FN-219 (0402) (October, 1970).

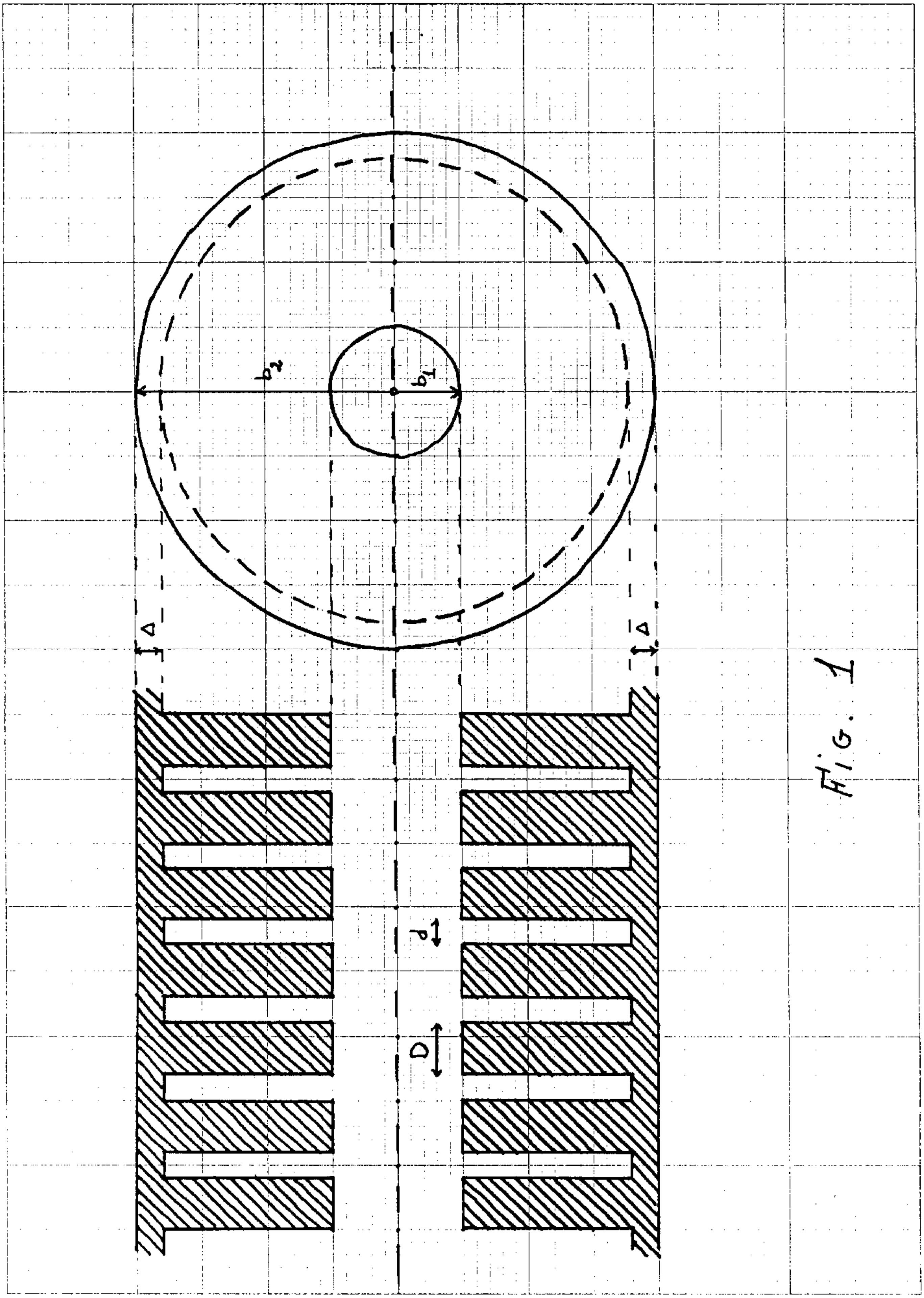

FIG. 1

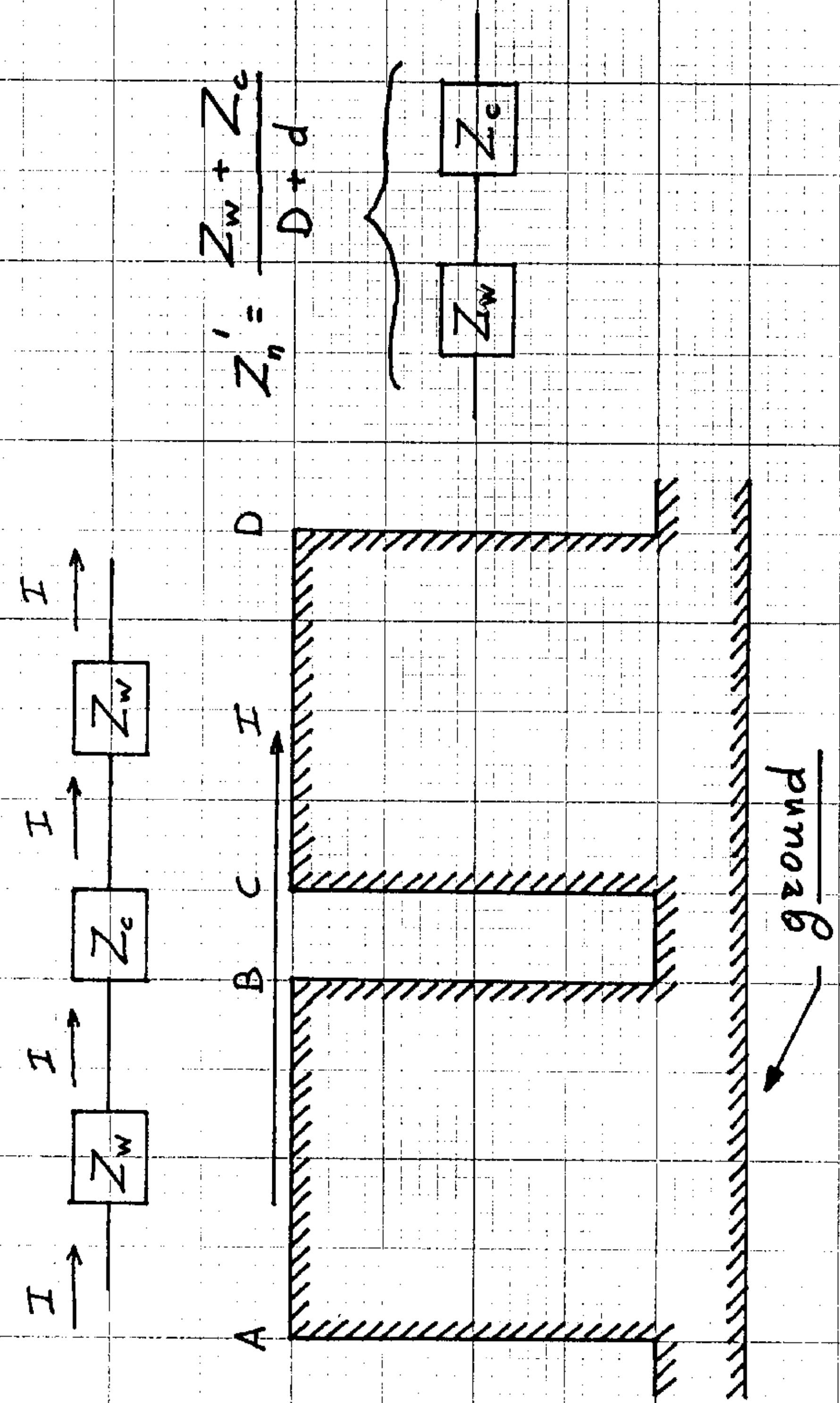

FIG. 2

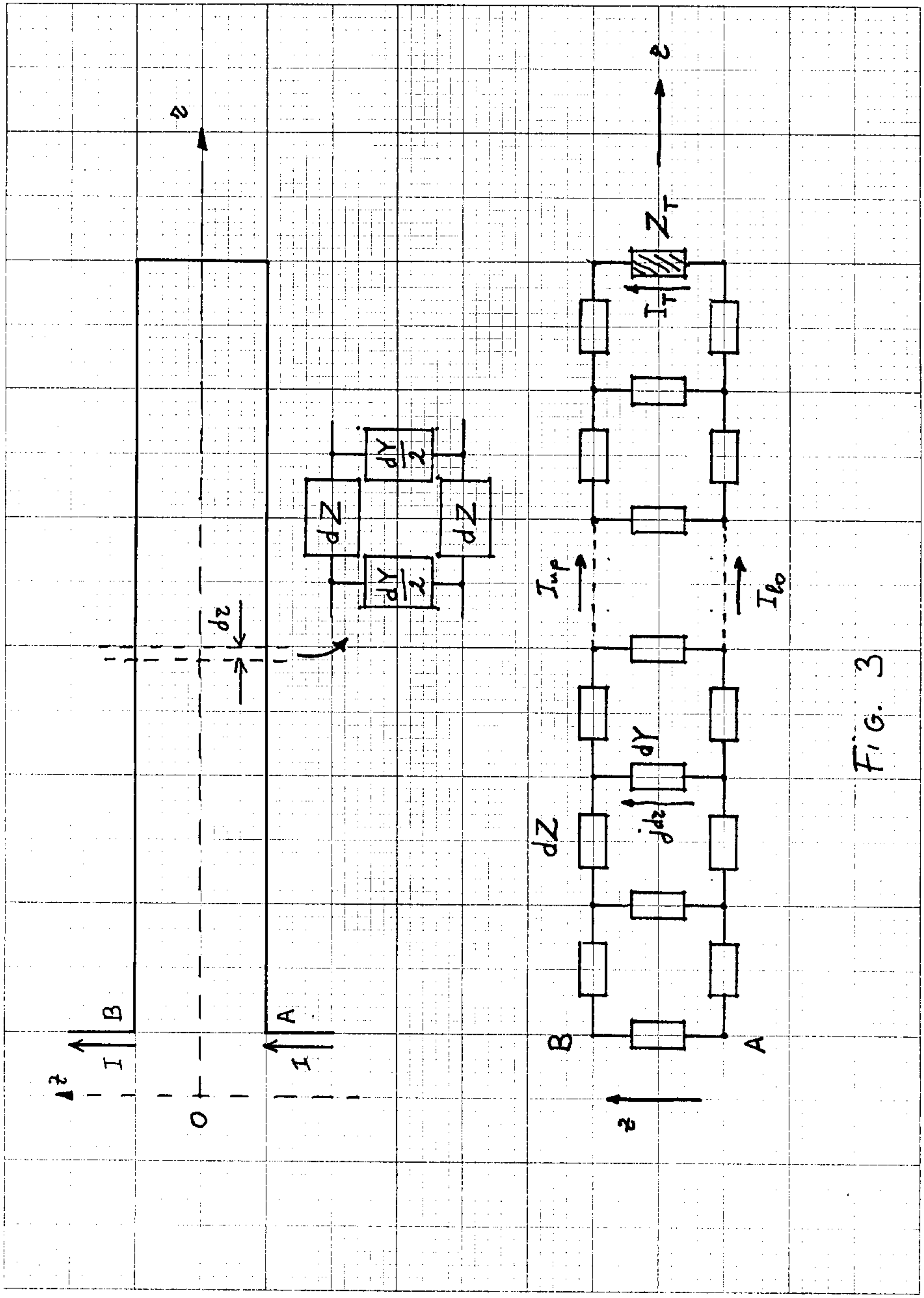

FIG. 3

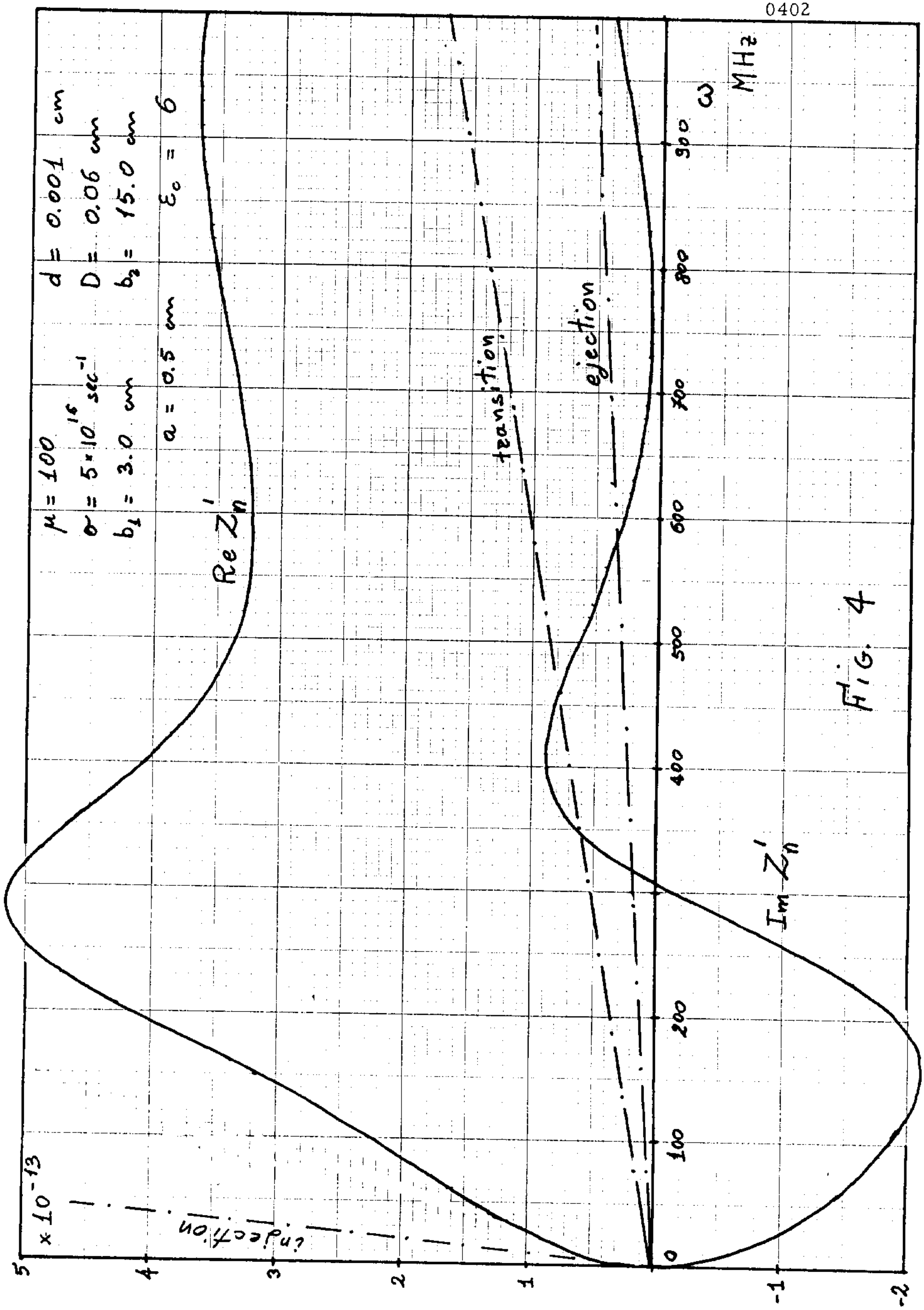
μ = 100
σ = 5×10^16 sec^-1
b1 = 3.0 cm
a = 0.5 cm
d = 0.001 cm
D = 0.06 cm
b2 = 15.0 cm
ε0 = 6
Re Z'n
Im Z'n
transition
ejection
injection
ω MHz
×10^-13
5
4
3
2
1
-1
-2
0
100
200
400
500
600
700
800
900

FIG. 4

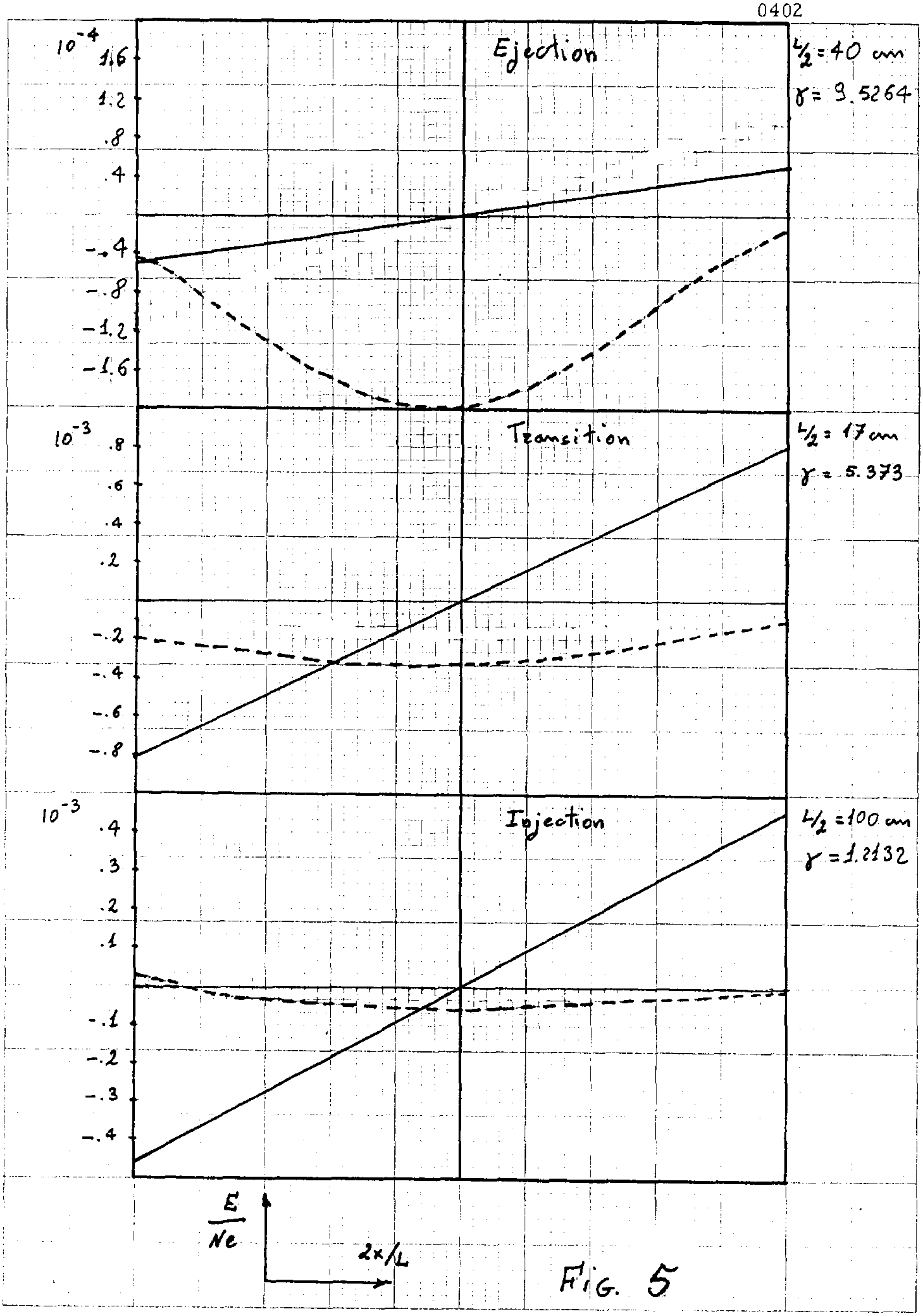
10^{-4}
1.6
1.2
.8
.4
-.4
-.8
-1.2
-1.6
Ejection
L/2 = 40 cm
γ = 9.5264
10^{-3}
.8
.6
.4
.2
-.2
-.4
-.6
-.8
Transition
L/2 = 17 cm
γ = 5.373
10^{-3}
.4
.3
.2
.1
-.1
-.2
-.3
-.4
Injection
L/2 = 100 cm
γ = 1.2132
E/Ne
2x/L

FIG. 5